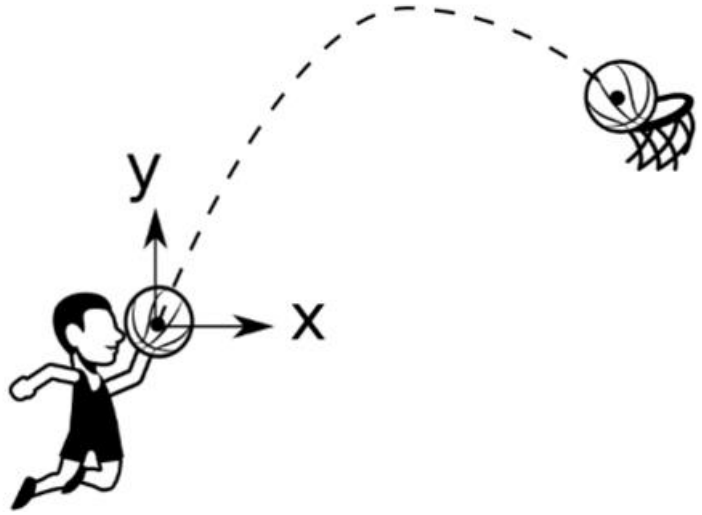

Projectile Motion

Mechanics-IAS Physics

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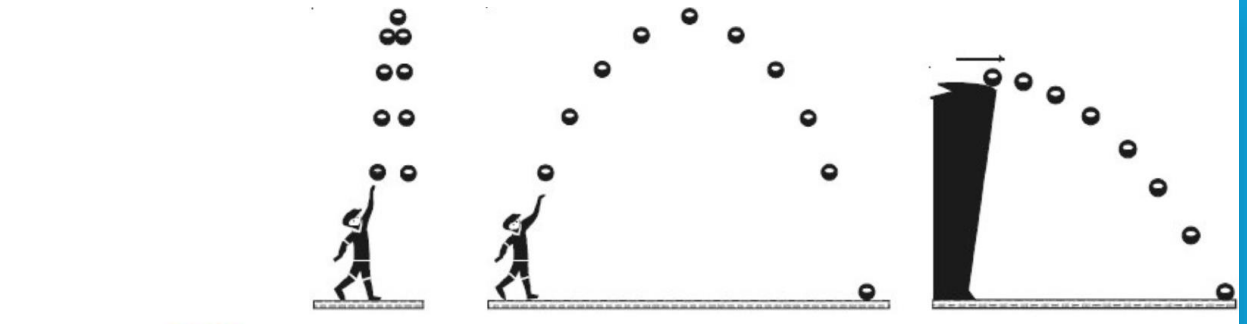
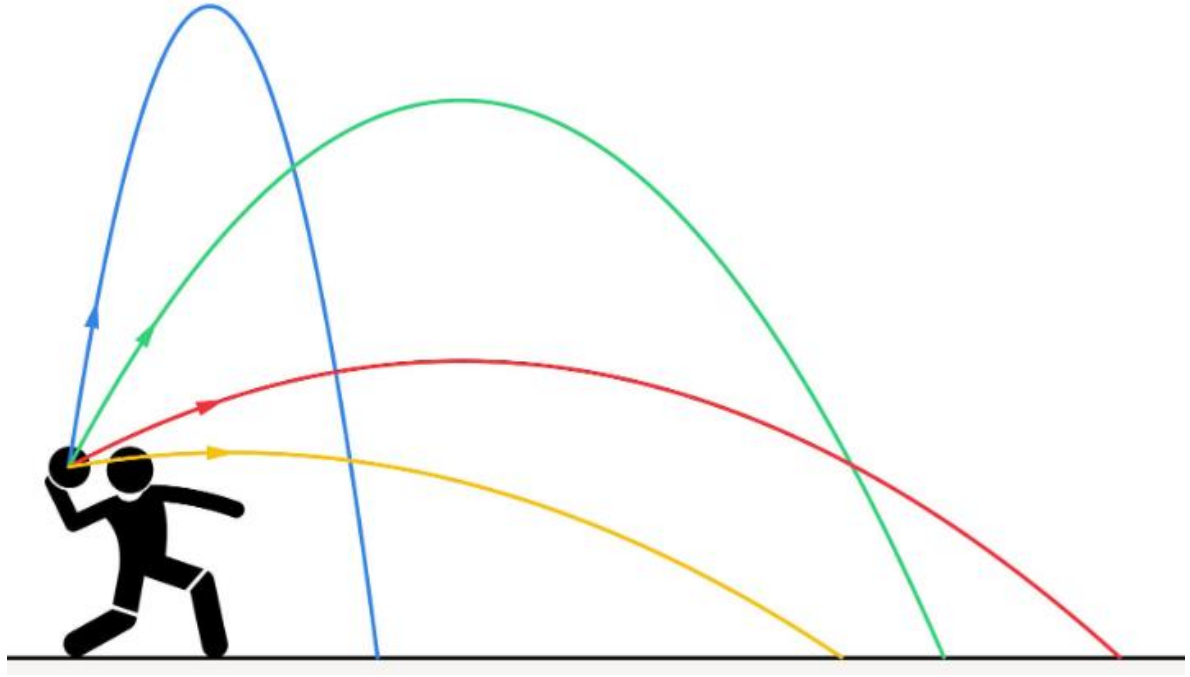




Objectives:

- Apply kinematics equations to moving objects.
- Apply the independence of horizontal and vertical motion to objects moving freely under gravity.
- Combine horizontal and vertical motion to calculate the movements of projectiles.

Equations of Motion

- $s = displacement,$
- $u = initial\ velocity,$
- $v = final\ velocity,$
- $a = acceleration,$
- $t = time$

$$s = \frac{(u + v)t}{2}$$

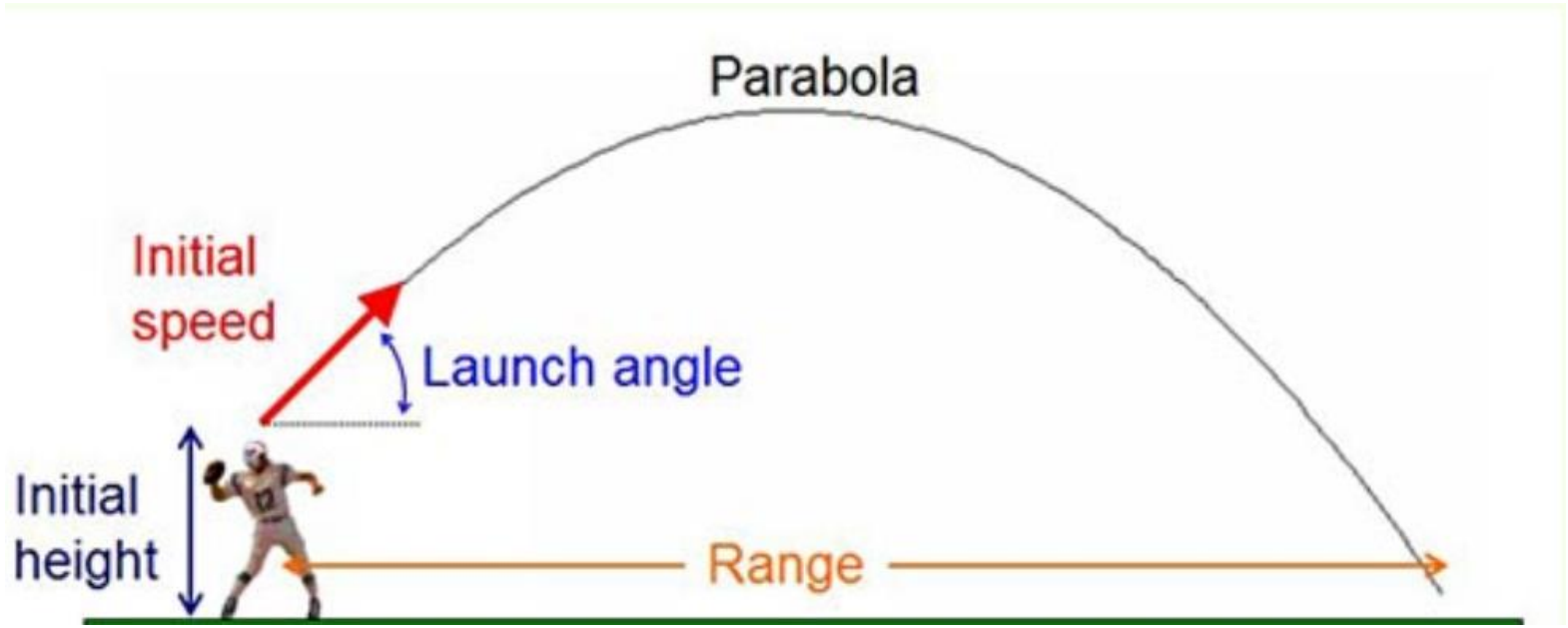
$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Projectile motion.

- Projectile motion refers to the motion of an object thrown or projected into the air, subject only to the acceleration due to gravity. It's a classic example of two-dimensional motion, involving both horizontal and vertical components.



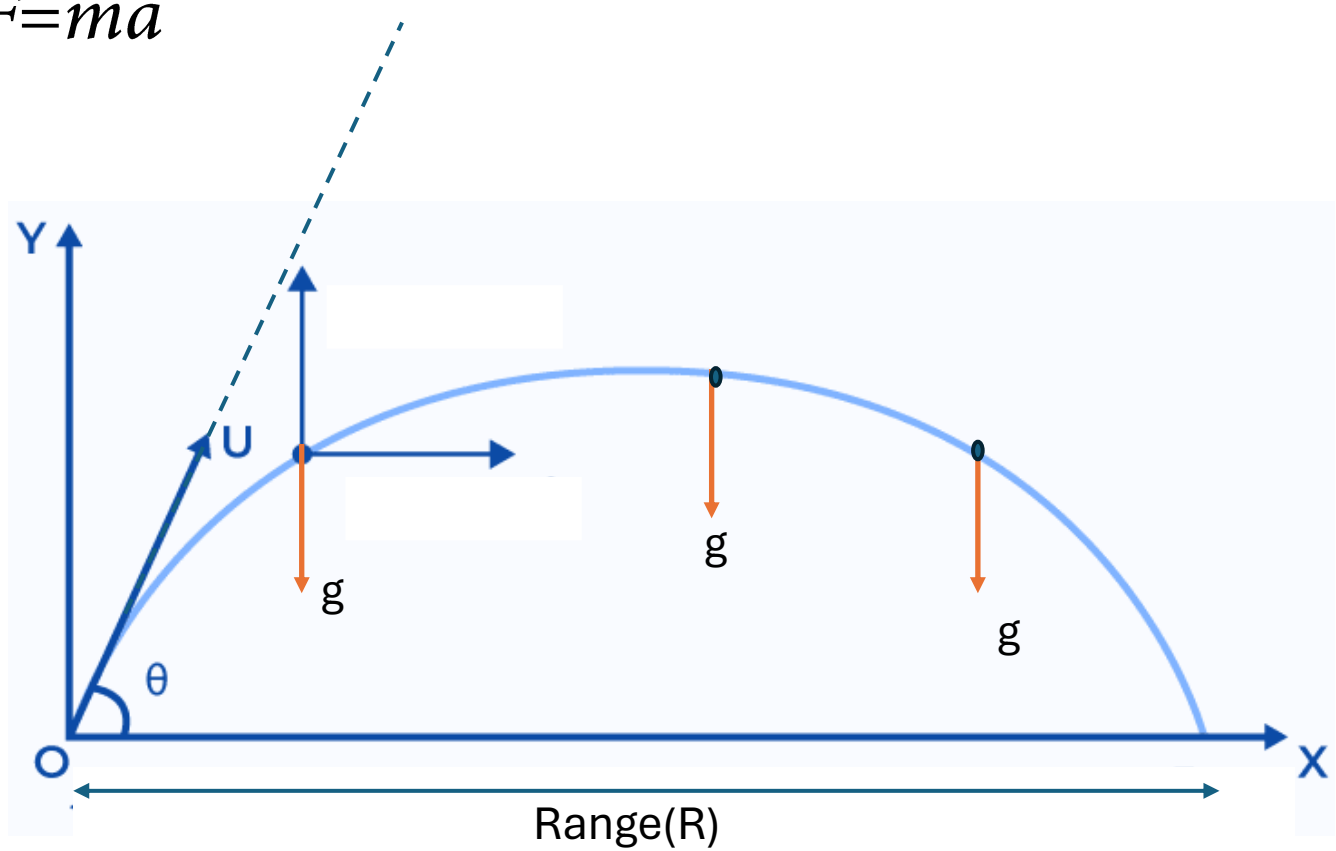
What is trajectory?

- A trajectory is the path followed by an object which is moving under given forces.
- For a projectile thrown near the surface of the earth, trajectory is **parabolic.**

- **Newton's first law** states that an object will remain at rest or in uniform motion in a straight line unless acted upon by an external force.

- Newton’s second law:

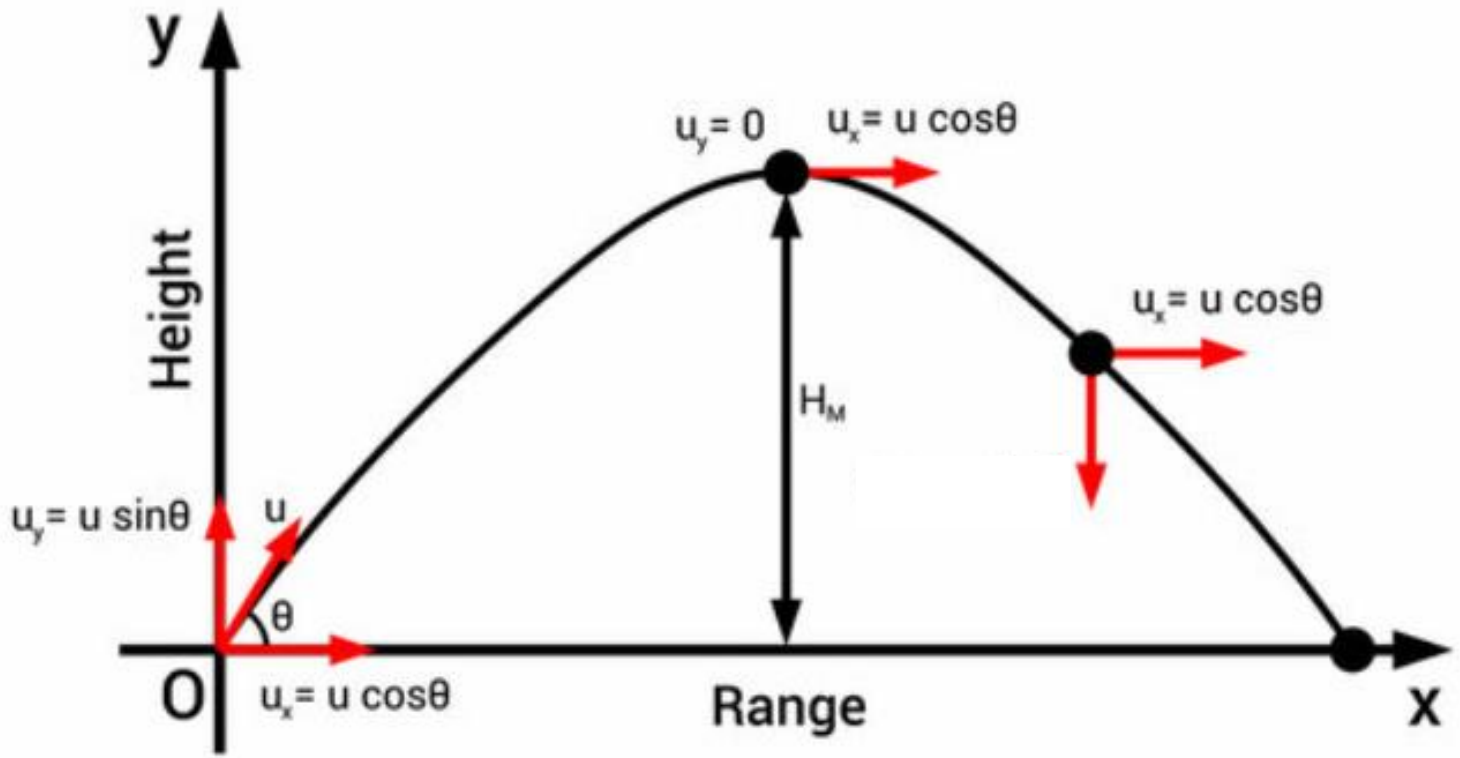
$$\sum F = ma$$



- Projectile motion can be broken down into two independent components: horizontal motion and vertical motion.
- Horizontal motion is constant and unaffected by gravity.
- Vertical motion is accelerated downward due to gravity.

Independence of Motion:

The horizontal and vertical motions of a projectile are independent of each other. This means that the horizontal velocity remains constant throughout the motion, while the vertical velocity changes due to acceleration.



Horizontal motion

- Initial velocity = $ucos\theta$
- Acceleration = 0
- Range (R)
- Velocity after time t,

$$ucos\theta$$

Vertical motion.

- Initial velocity = $usin\theta$
- Acceleration = -g
- Time of flight (T)
- Maximum Height (H)
- Velocity after time t,

$$usin\theta - gt$$

Time of flight(T)

As it hits the ground again the displacement in the vertical direction is 0.

$s = 0,\ intial\ vertical\ velocity = usin\theta$

$Time\ of\ flight = T$

$$\boldsymbol{T = \frac{2usin\theta}{g}}$$

$$\uparrow s = ut + \frac{1}{2}at^2$$

$$0 = (usin\theta)T - \frac{1}{2}gT^2$$

$$0 = T\left(usin\theta - \frac{1}{2}gT\right)$$

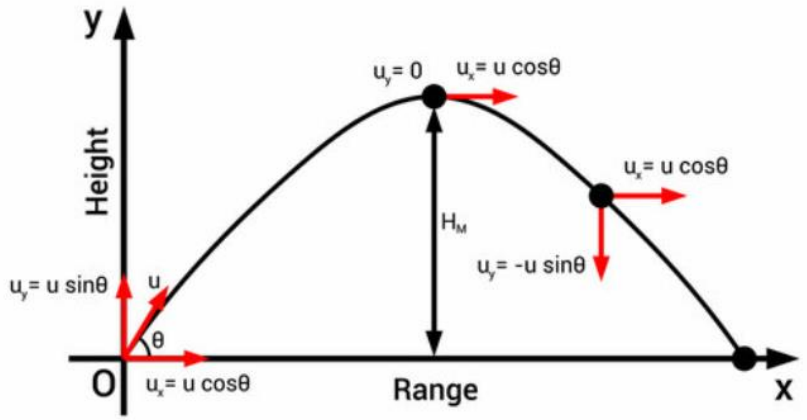
$$T = 0\ , \qquad T = \frac{2usin\theta}{g}$$

Range(R)

As it hits the ground,

$$\overrightarrow{s} = ut + \frac{1}{2}at^2$$

$$\overrightarrow{a} = 0\,, \qquad t = T$$



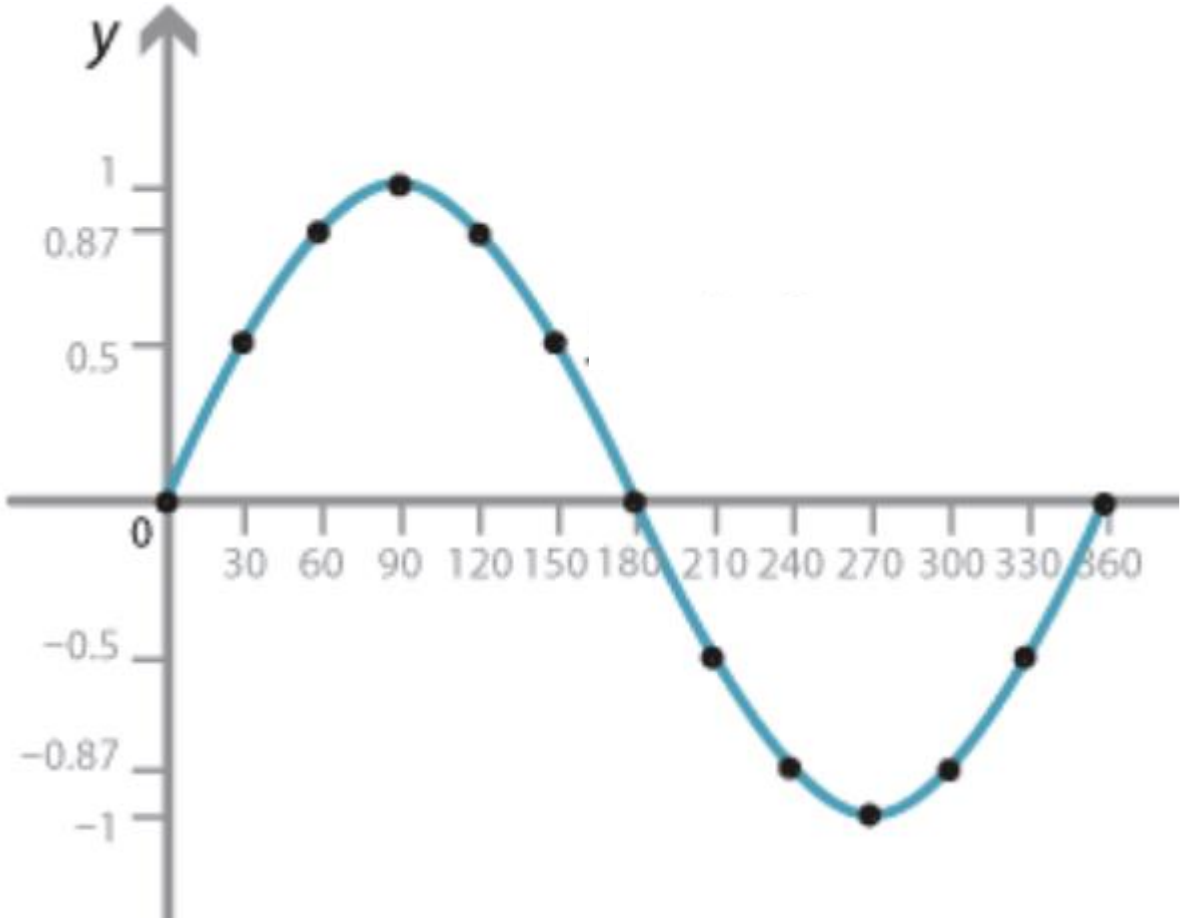
$$\boldsymbol{T = \frac{2usin\theta}{g}}$$

$$\overrightarrow{R} = ucos\theta \times \frac{2usin\theta}{g}$$

$$R = \frac{u^2}{g} sin2\theta$$

$$\boldsymbol{2sin\theta.\cos\theta = sin2\theta}$$

Maximum Range



$$R(max) = \frac{u^2}{g}(sin2\theta)_{max}$$

$$1 = (sin2\theta)$$

$$sin\ 90^0 = (sin2\theta)_{max}$$

$$90^0 = 2\theta$$

$$45^0 = \theta$$

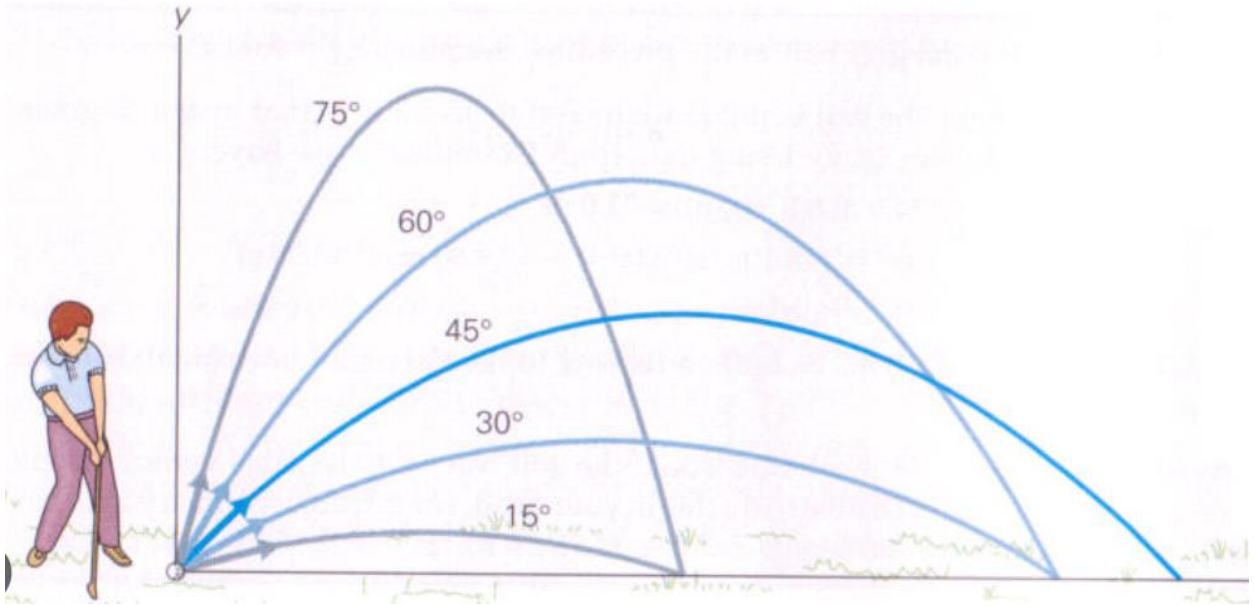
Special case: $\propto = 90^0 - \theta$

$$R = \frac{u^2}{g} \sin(2 \propto)$$

$$R = \frac{u^2}{g} sin2(90^0 - \theta)$$

$$R = \frac{u^2}{g} sin(180^0 - 2\theta)$$

$$R = \frac{u^2}{g} sin2\theta$$



$sin(180^0 - \beta) = \sin\beta$

Maximum height(H)

At maximum height, vertical velocity = 0

$$v^2 = u^2 + 2as$$
$$0 = (usin\theta)^2 - 2gH$$

$$H = \frac{u^2}{2g}\sin^2\theta$$

Time to reach maximum height.

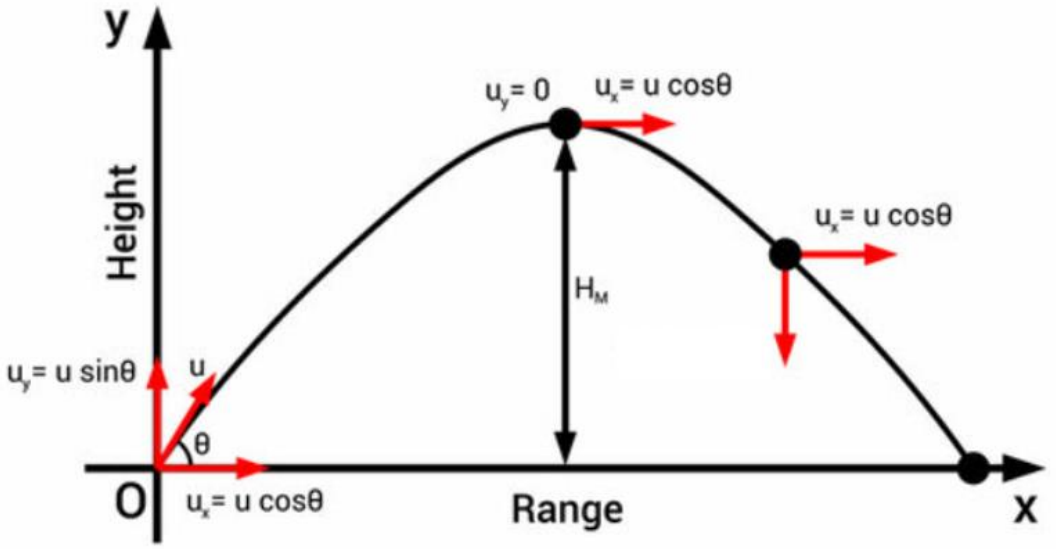
At max height vertical velocity =0

$$v = u + at$$
$$0 = usin\theta - gt$$

$$t = \frac{usin\theta}{g}$$

$$\boldsymbol{T = \frac{2usin\theta}{g}}$$

$$t = \frac{T}{2}$$



Quiz

1) What is the path followed by a projectile under the influence of gravity alone?

a. Straight line b. Parabola c. Circle d. Ellipse

2) At the highest point of a projectile's trajectory, which component of its velocity becomes zero?

a. Horizontal velocity b. Vertical velocity

c. Total velocity d. None of the above

3) What is the angle of projection for a projectile launched with the maximum range under the same initial velocity?

a. 30 degrees b. 45 degrees c. 60 degrees d. 90 degrees

4) Which of the following factors affects the time of flight of a projectile?

a. Initial velocity b. Angle of projection

c. Height of projection d. All of the above

5) If two projectiles are launched horizontally from the same height but with different initial velocities, which one will reach the ground first?

a. The one with higher initial velocity b. The one with lower initial velocity

c. Both will reach the ground at the same time d. It depends on the angle of projection.

Concept Learning Questions

(g=9.81 m/s^2)

1) A ball is kicked from the ground with an initial velocity of 20 m/s at an angle of 30 degrees above the horizontal.

Calculate:

a. The horizontal component of its initial velocity.
b. The vertical component of its initial velocity.
c. The time it takes for the ball to reach its maximum height.
d. The maximum height the ball reaches.
e. The total time the ball is in the air.
f. The horizontal distance the ball travels before hitting the ground.

Summary

Yes/No(if yes-direction?)	Horizontal Motion	Vertical Motion
Forces present?	No	Yes-weight-act downwards
Acceleration present?	No	g=9.81 m/s^2 downwards
Velocity constant or changing?	constant	Changing by 9.81 m/s each second

The horizontal and vertical motions of a projectile are independent of each other.

$$\boldsymbol{T = \frac{2usin\theta}{g}} \qquad R = \frac{u^2}{g} sin2\theta \qquad H = \frac{u^2}{2g} \sin^2 \theta$$