5B Circular Motion

Advanced Level-A2 Physics

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Radian (rad)

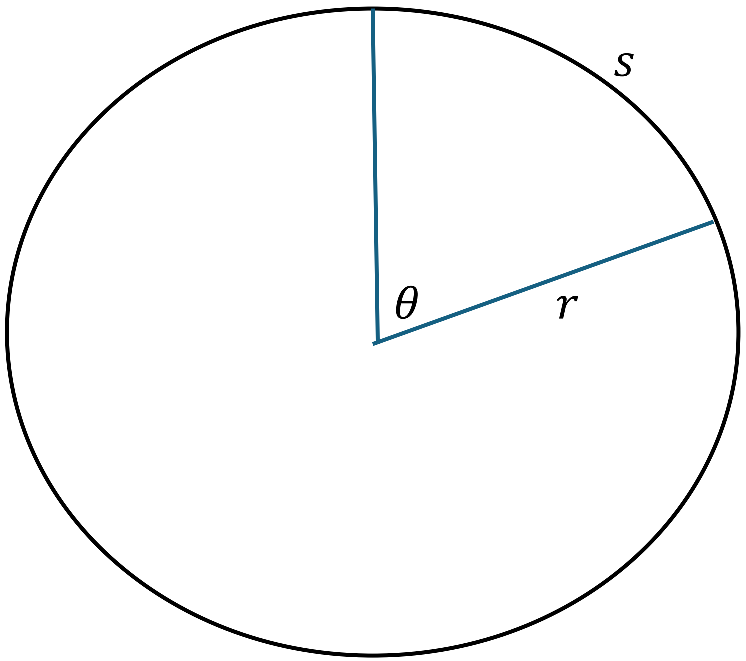
Angle(in radians) $= \frac{length\ of\ arc}{radius\ of\ arc} = \frac{s}{r}$

$\theta = \frac{2\pi r}{r}\ 2\pi\ rad$

$360^0 = 2\pi\ rad$

$180^0 = \pi\ rad$

If the arc of a circle equals the radius, then the angle subtended by that arc at centre of the circle is called a radian; it is equal to about 57.3^0.



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Concept Learning Questions

Write the following angles in radians.

$a)\ 30^0$

$b)\ 60^0$

$c)\ 270^0$

$d)\ 90^0$

Concept Learning Questions

Write the following angles in degrees.

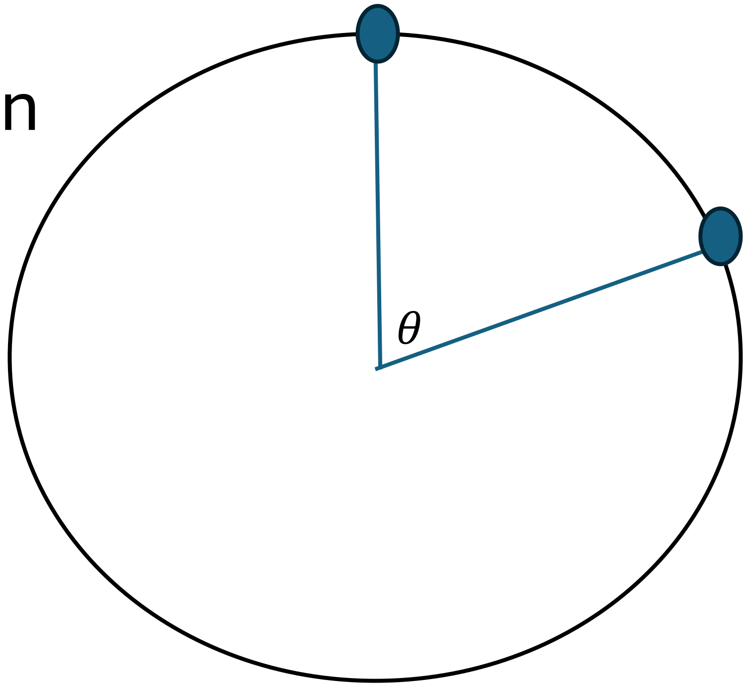
$a)\ \ \pi\ rad$

$b)\frac{2\pi}{3}\ rad$

c) $\frac{3\pi}{4}\ rad$

Angular displacement($\Delta\theta$)

- Angular displacement is the vector measurement of the angle through which something has moved.
- Positive direction- Anticlockwise direction
- SI unit- radians (rad)



Angular velocity(ω)

- The rate at which the angular displacement changes is called angular velocity.
- Angular velocity = $\frac{Angular\ displacement}{time\ taken}$

$$\boldsymbol{\omega = \frac{\Delta\theta}{\Delta t}}$$

If the object completes full cycle $\Delta\theta = 2\pi\ radians$ in a time period T. Then the angular velocity given by:

$$\boldsymbol{\omega = \frac{2\pi}{T}}$$

$$\boldsymbol{f = \frac{1}{T}}$$

$$\boldsymbol{\omega = 2\pi f}$$

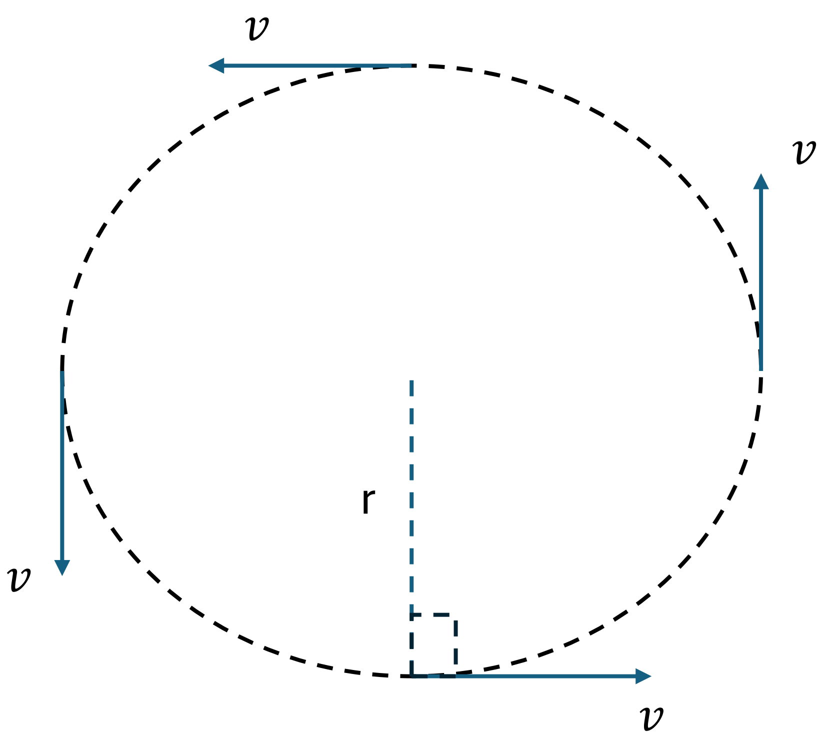
Concept Learning Questions.

1) A geostationary satellite orbiting the Earth. What is the angular velocity ?

(Ans: 7.27 x $10^{-5}\ rad\ s^{-1}$)

2) What is the angular velocity of an athletic hammer if the athlete spins it at a rate of 4 revolutions per second ?

Instantaneous velocity (v)



$$\boldsymbol{v = r\omega}$$

The linear velocity of the object at a point is called instantaneous velocity.

$$v = \frac{s}{t}$$

$$s = r\theta$$

$$v = \frac{r\theta}{t} = r\frac{\theta}{t} = r\omega$$

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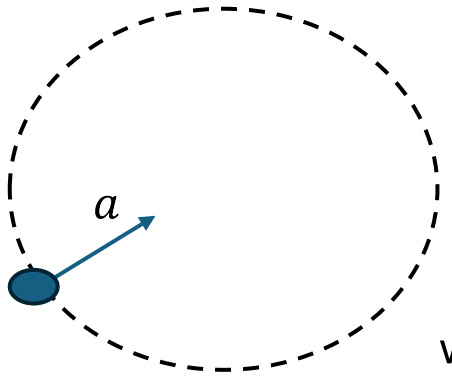
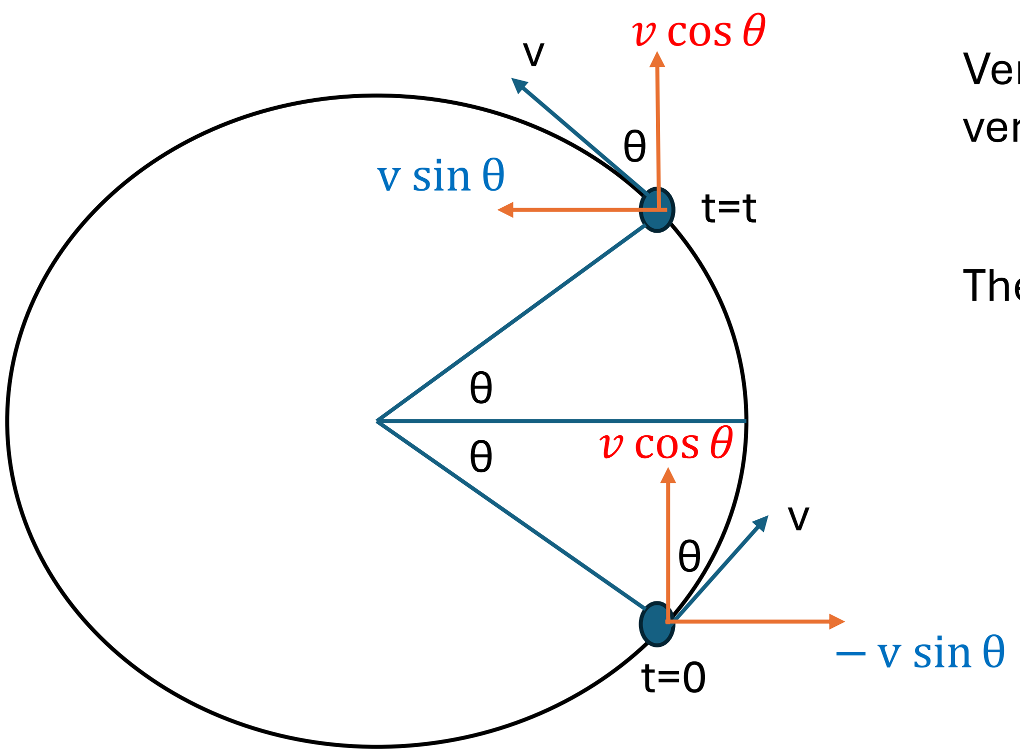
Centripetal acceleration(a)

- If a body moves in a circle at a constant speed, it is said to be in uniform circular motion. In such a motion, the magnitude of the velocity(i.e. speed) is constant but the direction of the velocity is changing with time. Hence the motion of the body is accelerated.

- The acceleration is directed towards the centre of the circle and is called centripetal acceleration.



Centripetal acceleration(a)



Vertical component of the velocity is the same. The vertical acceleration is zero.

The acceleration in the horizontal direction;

$$a = \frac{v\sin\theta - (-v\sin\theta)}{t}$$

$$a = \frac{2v\sin\theta}{t} \qquad v = \frac{r(2\theta)}{t}$$

$$a = \frac{2v\sin\theta}{\frac{r(2\theta)}{v}} = \frac{v^2}{r}\frac{(\sin\theta)}{\theta}$$

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

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$$\boldsymbol{a = \frac{v^2}{r}}$$

$\mathbf{v} = \boldsymbol{r\omega}$

$$\boldsymbol{a = r\omega^2}$$

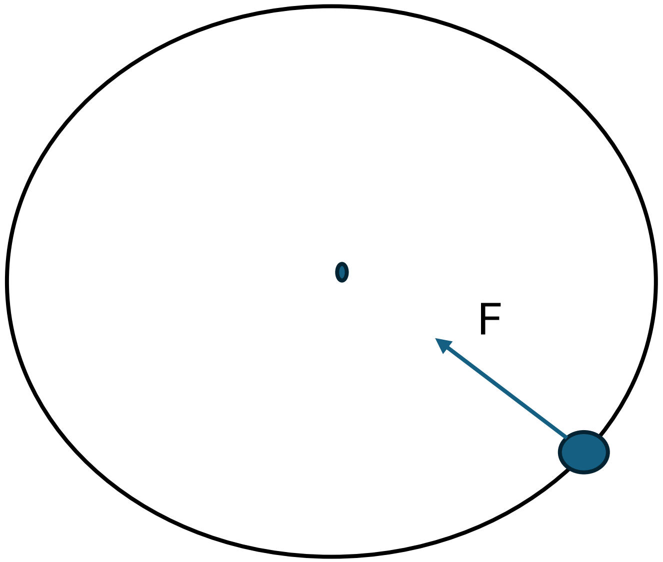
Centripetal force (F)

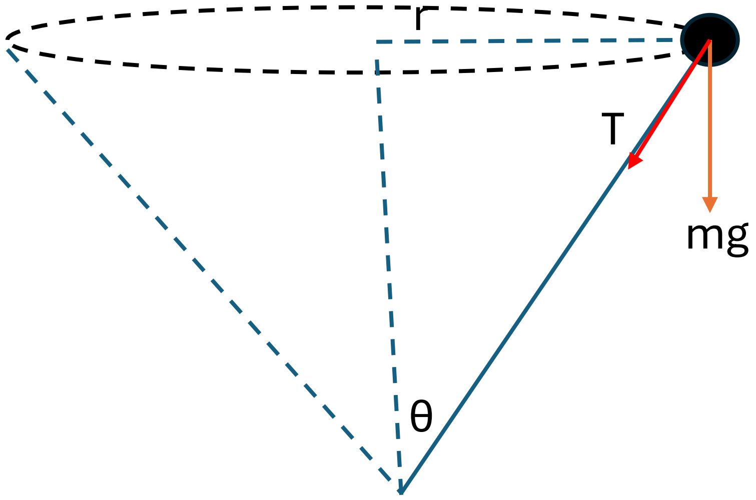
The resultant force acting towards the centre of the circular path.

F = ma

$$F = \frac{mv^2}{r}$$

$$F = mr\omega^2$$

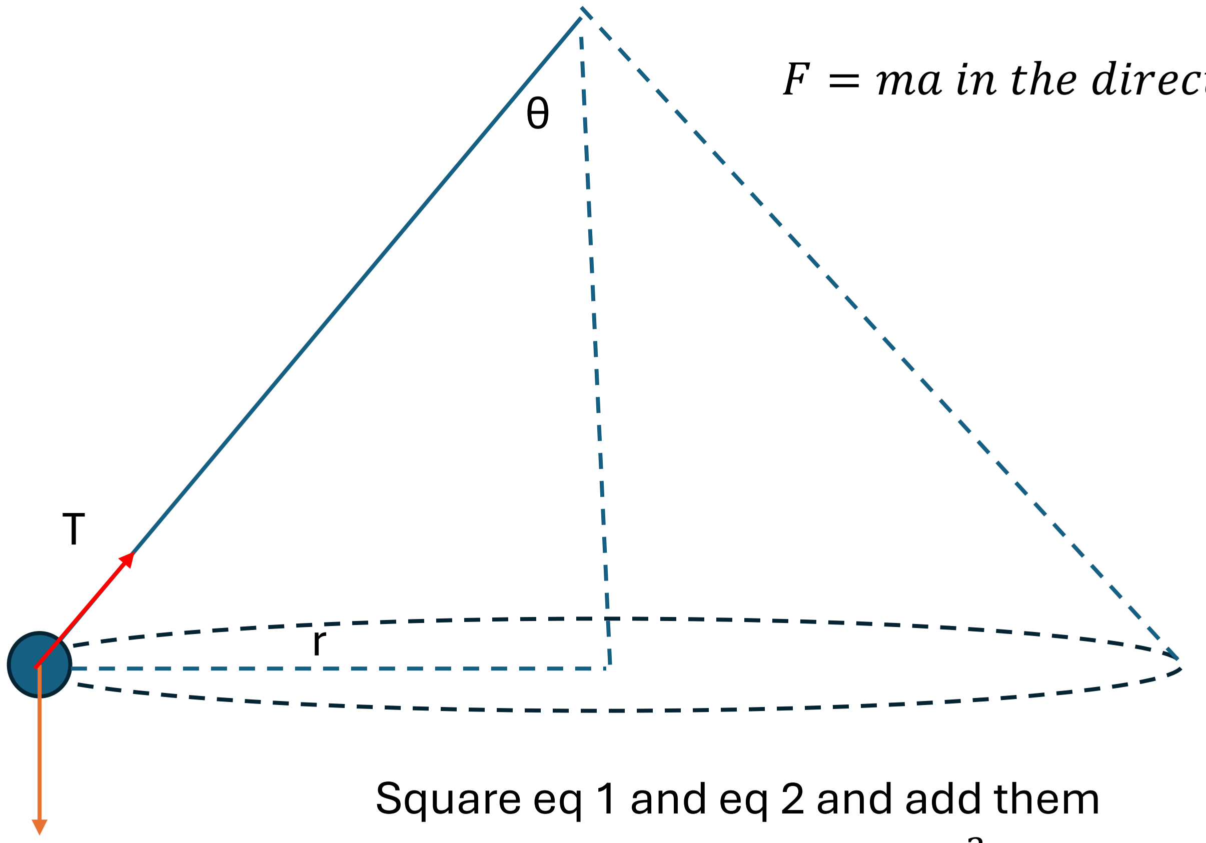




$$T \sin\theta = \frac{mv^2}{r}$$

Centripetal force is provided by the horizontal component of the string.

Motion in a horizontal circle.



$$F = ma \text{ in the direction of the centre of the circle}$$

$$T \sin\theta = \frac{mv^2}{r} \rightarrow 1$$

$$T \cos\theta = mg \rightarrow 2$$

Divide eq 1 by eq 2.

$$Tan\ \theta = \frac{v^2}{gr}$$

Square eq 1 and eq 2 and add them

$$T^2\underbrace{(\sin^2\theta + \cos^2\theta)}_{=1} = \left(\frac{mv^2}{r}\right)^2 + (mg)^2$$

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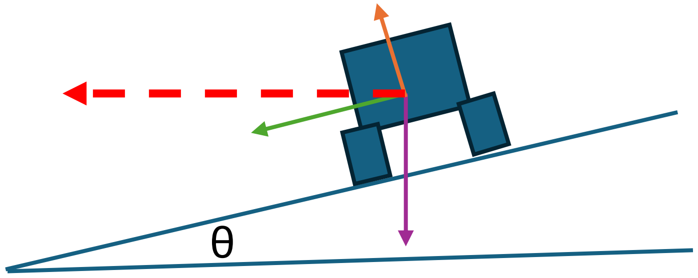
Examples:

- *When a car negotiates a curved level road, the centripetal force required to keep the car in motion around the curve is provided by the friction between road and the tyres.*

- *The large amount of friction between the tyres and the road would damage the tyres. To minimize the wearing out of tyres the road-bed is banked, i.e. the outer part of the road raised a little so that the road slopes towards the centre of the curved track.*



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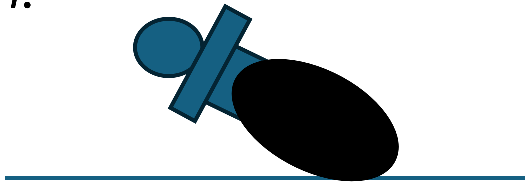
- *While negotiating a curved level road a cyclist has to lean inwards which provides the necessary centripetal force which prevent him from falling down.*



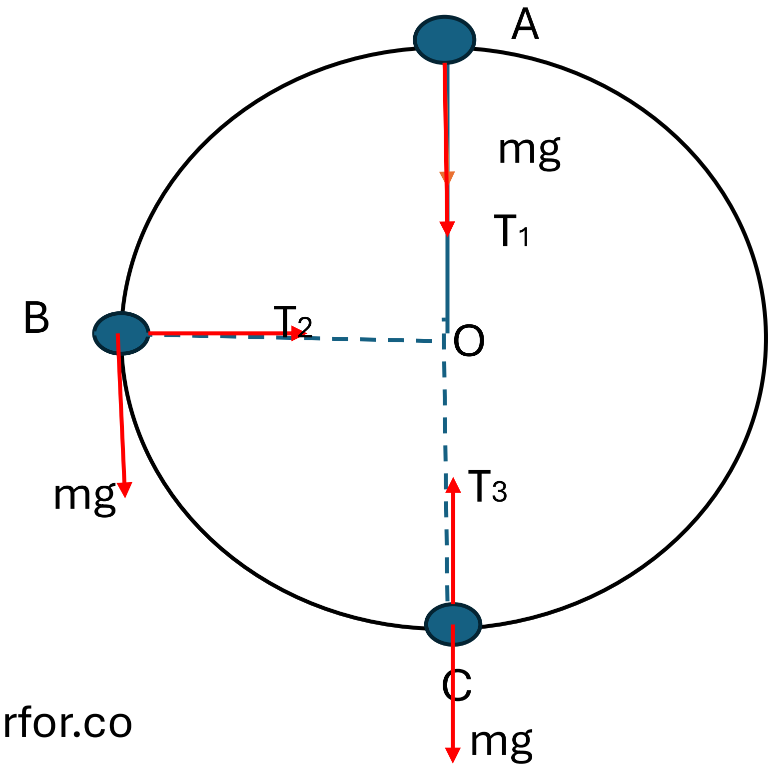
Motion in a vertical circle

The following figure shows an object of mass m whirled with a constant speed v in a vertical circle of centre O with a string of length r. When the object is at top A of the circle, let us say that the tension(force) in the string is T_1 . Since the weight mg acts vertically downwards towards the centre O, we have,

Force towards centre, $F = T_1 + mg = \frac{mv^2}{r}$

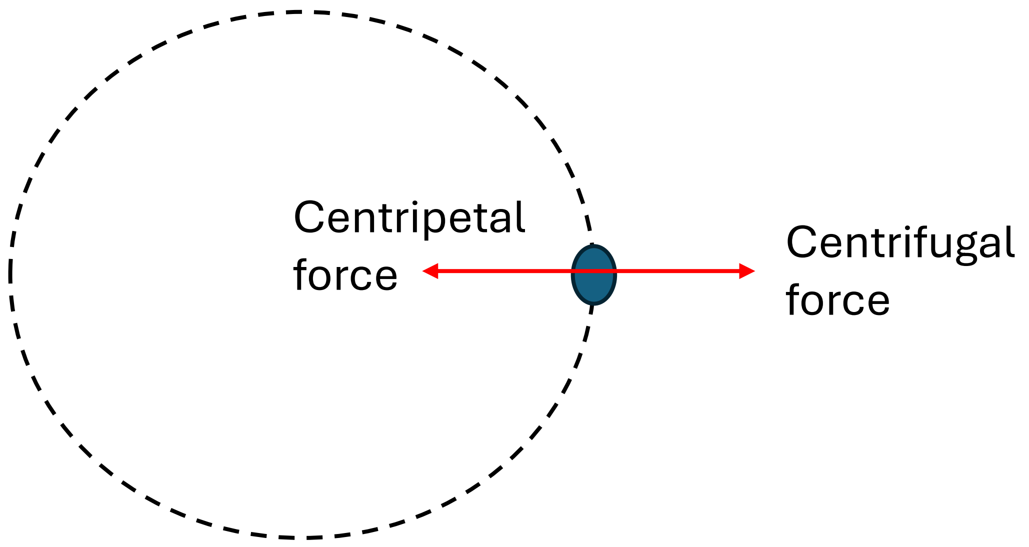
At B , $T_2 = \frac{mv^2}{r}$

At C, $T_3 - mg = \frac{mv^2}{r}$



Centrifugal force (pseudo force)

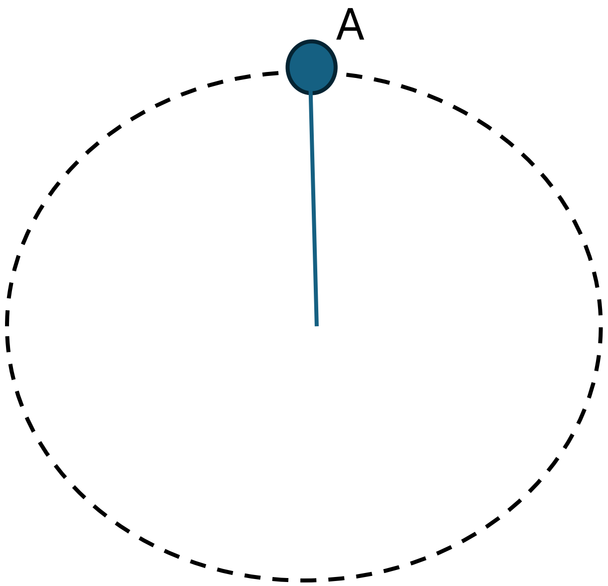
Centrifugal force is the apparent (outward) force that seems to push an object away from the centre of rotation when the object is moving along a circular path. It is perpendicular to the direction of movement.



Concept Learning Questions.

1) A string of length L =0.5 m is fixed at one end and carries mass of 200 g at the other end. The string makes $\frac{2\sqrt{3}}{\pi}$ revolutions per second about a vertical axis passing through its second end. What is the angle of inclination of the string with the vertical. ($g = 9.81\ m/s^2$)

2) One end of a string of length 1 m is tied to a body of mass 0.5 kg. It is whirled in a vertical circle as shown in the following figure. If the angular frequency of the body is 4 rad s^{-1} , what is the tension in the string when the body is at the topmost point A?



3) A string can withstand a tension of 25 N. What is the greatest speed at which a body of 1 kg can be whirled in a horizontal circle using a 1 m length of a string ?

4) A cyclist is moving with a speed of 6 m / s. As he approaches a circular turn on the road of radius 120 m, he applies brakes and reduces his speed at a constant rate 0.4 $m\ s^{-2}$. The magnitude of the net acceleration of the cyclist on the circular turn is,

a) 0.5 $m\ s^{-2}$
b) 1.0 $m\ s^{-2}$
c) 2.0 $m\ s^{-2}$
d) 4.0 $m\ s^{-2}$