6A –Electric Fields

International Advanced Level Physics-IA2

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Electric Field

A space that will cause charge particles to accelerate is said to have an electric field.

Electric field strength: Electric force per unit charge.

$$\boxed{E = \frac{F}{Q}}$$

Unit of E : NC^{-1}

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F=EQ

Using Newton's second law F=ma

$$F = EQ$$

$$F = ma$$

$$\boxed{a = \frac{EQ}{m}}$$

Concept Learning Questions

What acceleration will an electron experience when it is an electric field generated by an X-ray machine's electric field which has strength of 4.5 x $10^5\ Vm^{-1}$?

(mass of electron = 9.11 x$\mathbf{10^{-31}}\ \boldsymbol{kg, e = -1.6\ \times 10^{-19}\ C}$)

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Electric Potential(V)

Electric potential energy per unit charge is called electric potential.

$$\boxed{V = \frac{E_{\mathrm{p}}}{Q}}$$

In an electric field, the kinetic energy gained by a charge moving within the field is given by,

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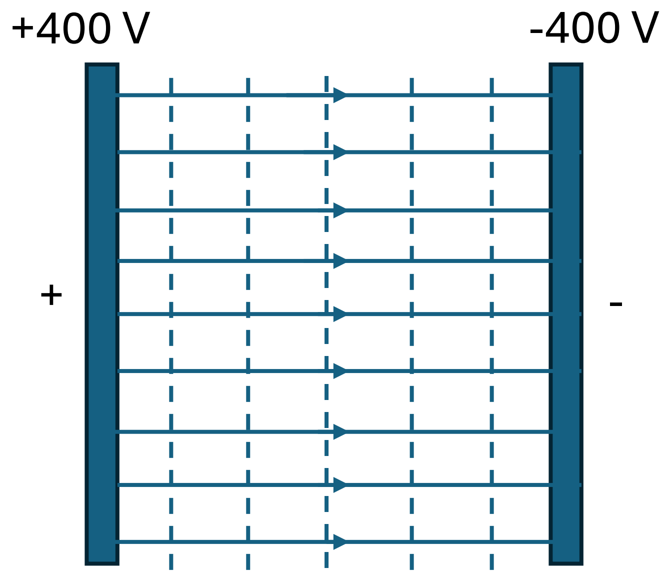
$$\boxed{E_{\mathrm{k}} = VQ}$$

Uniform fields



Equipotentials.

As we move through an electric field, the electrical potential changes from place to place. Those locations that all have the same potential can be connected by lines called equipotentials.

The strength of a uniform electric field is a measure of how rapidly it changes the potential over distance.

$$\boxed{E = \frac{V}{d}}$$

Unit of Electric field strength: $N\ C^{-1}$ or $V\ m^{-1}$

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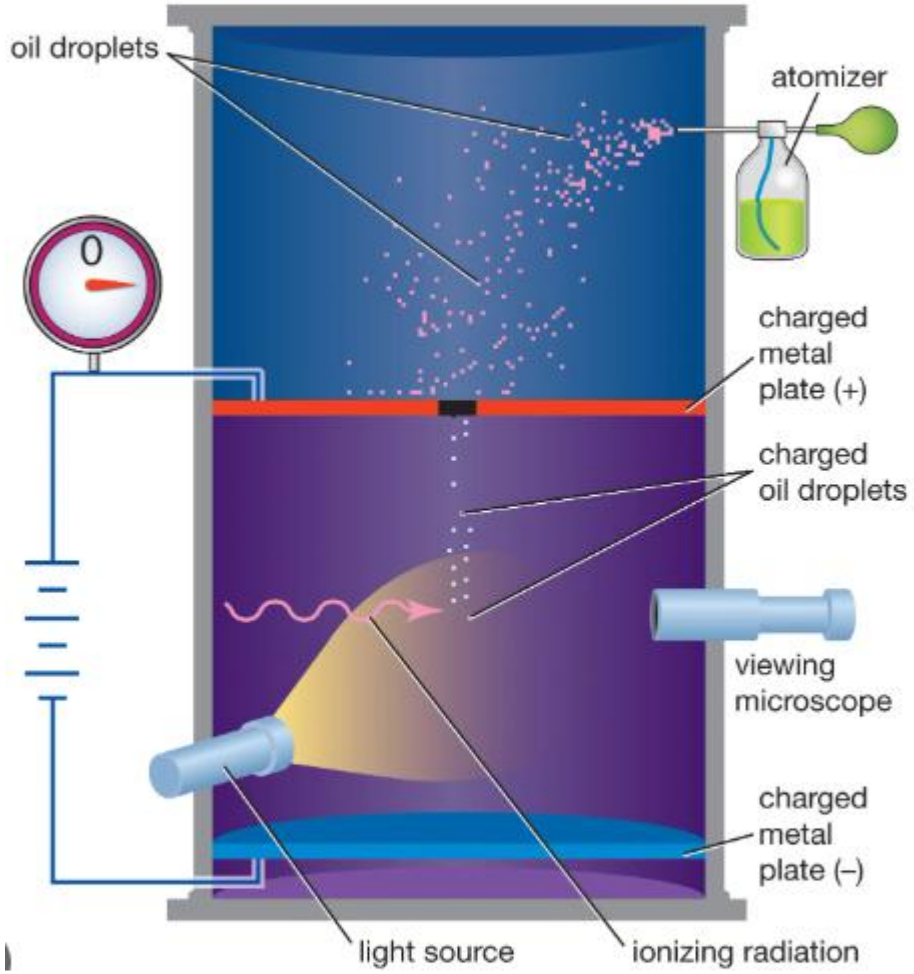
Concept Learning questions.

- In an X-ray machine there is a potential difference of 50 000 V between a cathode and an anode. These electrode plates are 20 cm apart. What is the electrical field strength between the plates?

Millikan’s oil drop experiment



Millikan's oil drop experiment, conducted by Robert A. Millikan in 1909, determined the charge of the electron. In this experiment, tiny oil droplets were sprayed into an electric field between two charged plates. By adjusting the voltage, Millikan was able to suspend individual droplets in mid-air. Using Stokes' law and the balance of gravitational and electric forces, he calculated the elementary charge of an electron to be approximately 1.6×10^{-19} coulombs, confirming the quantization of electric charge.

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Concept Learning Questions.

The terminal velocity v of an oil droplet was measured as it fell a known distance in air when the plates were uncharged. Stokes' law was then used to determine the radius r of the oil droplet. Upthrust was ignored.

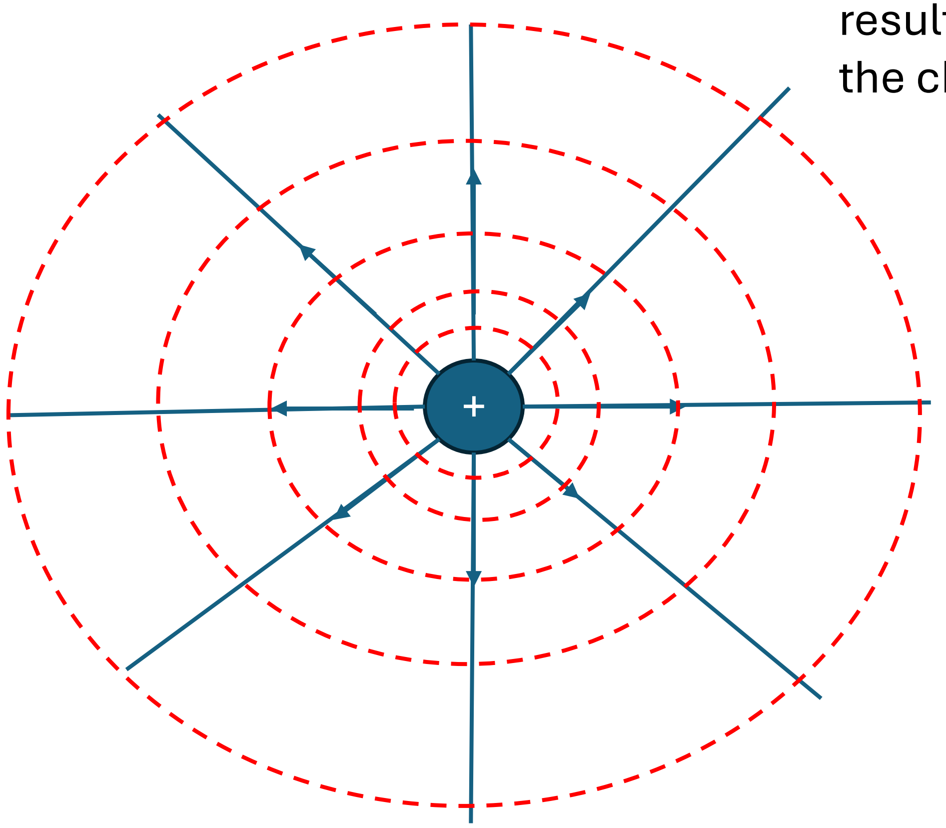
Show that $r = \sqrt{\dfrac{9\eta v}{2\rho g}}$

Derive an expression for the charge of oil drop.

An oil drop of mass 1.6×10^{-15} **kg** is suspended between two parallel plates with an electric field of 3.2×10^4 **V/m**. Calculate the charge on the oil drop.

(Take $g = 9.81\, m/s^2$)

Radial Electric fields



A charged particle inside a charged sphere would experience no resultant force and so there is no electric field. The overall effect of all the charges on the sphere cancel out within the sphere itself.

Gauss's law states that the total electric flux through a closed surface is:

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q}{\varepsilon_0}$$

For a **spherically symmetric charge distribution**, consider a Gaussian surface of radius r around the charge Q. Since the electric field is radial and uniform over the sphere, we get:

$$E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

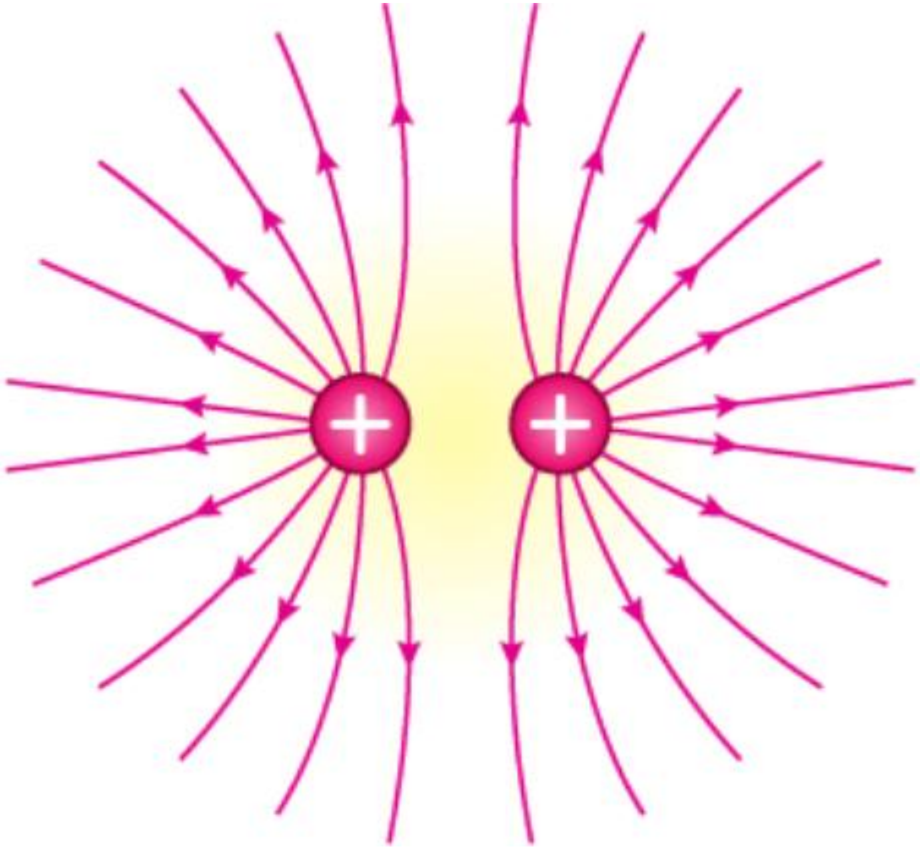
Solving for E:

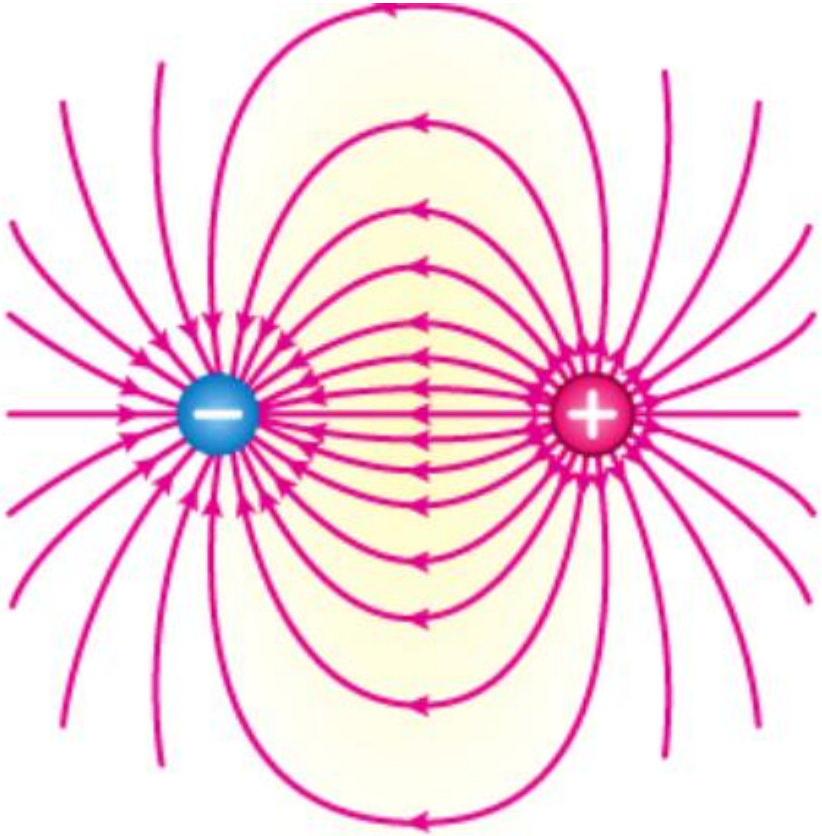
$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

Concept Learning Questions.

What is the electric field strength at a distance of $2 \times 10^{-8}\ m$ from a proton?

Draw equipotential lines on the following diagrams.





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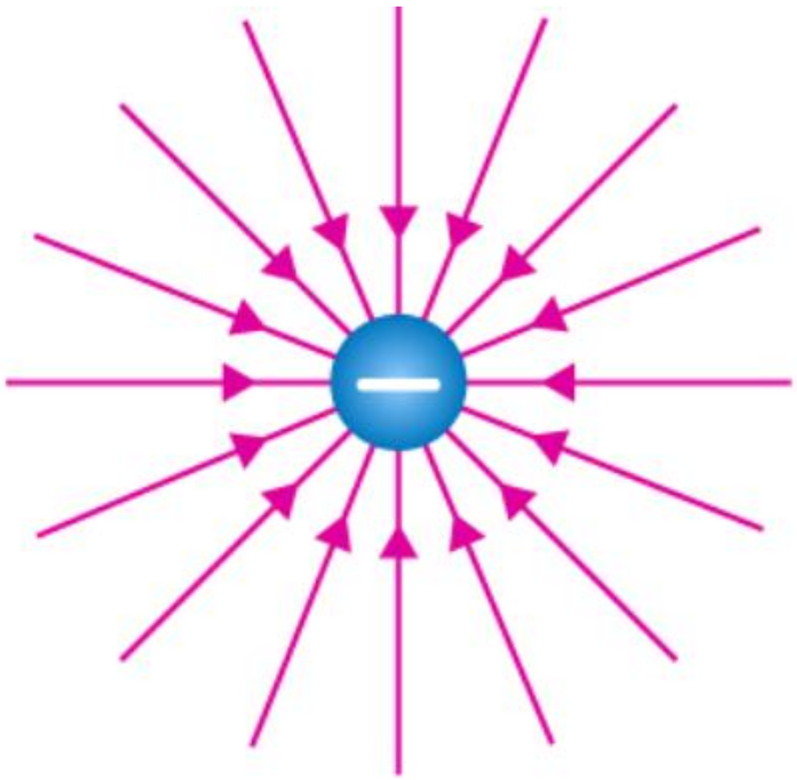

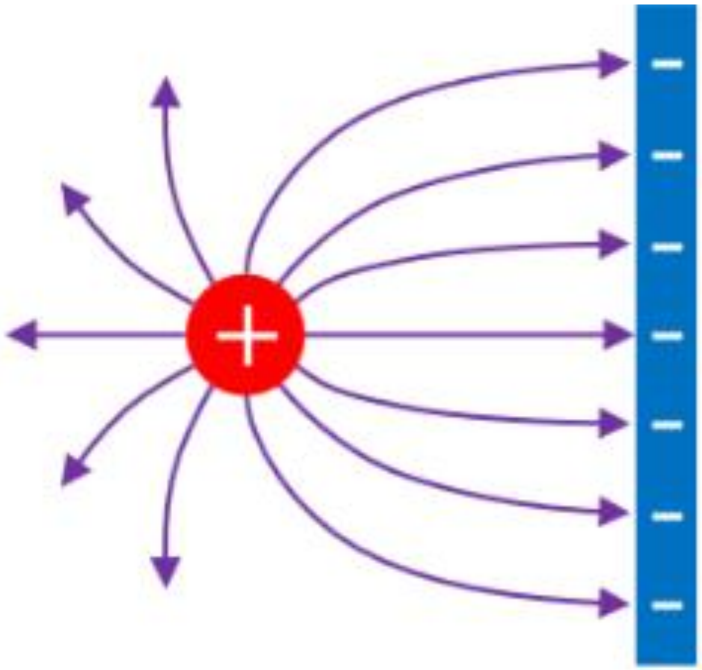

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Concept Learning Questions

A charge of 5×10^{-6} C is placed in vacuum. Find the electric field at a distance of **0.2 m** from the charge.

(Take $\varepsilon_0 = 8.85 \times 10^{-12}\, C^2/N \cdot m^2$)

Potential in a radial electric field

Electric field E is the negative gradient of potential V:

$$E = -\frac{dV}{dr}$$

Rearranging for dV:

$$dV = -E\,dr$$

From **Coulomb's law**, the radial electric field at a distance r from a point charge Q is:

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Substituting into dV:

$$dV = -\frac{Q}{4\pi\varepsilon_0 r^2}dr$$

To find the potential at a distance r, we integrate from some reference point (usually infinity, where $V = 0$):

$$V = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}dr$$

Evaluating the integral:

$$V = -\frac{Q}{4\pi\varepsilon_0}\int_{\infty}^{r} \frac{1}{r^2}dr$$

$$V = -\frac{Q}{4\pi\varepsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$\boxed{V = \frac{Q}{4\pi\varepsilon_0 r}}$$

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Coulomb's constant k is related to the permittivity of free space ε_0 by:

$$k = \frac{1}{4\pi\varepsilon_0} \approx 9.0 \times 10^9 \, N \cdot m^2/C^2$$

Using **Coulomb's constant**, the equations can be rewritten as follows:

1. Radial Electric Field Strength

$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{kQ}{r^2}$$

2. Electric Potential Due to a Point Charge

$$V = \frac{Q}{4\pi\varepsilon_0 r} = \frac{kQ}{r}$$

3. Work Done in Moving a Charge

Work done in moving a charge q from r_1 to r_2:

$$W = q(V_1 - V_2) = q\left(\frac{kQ}{r_1} - \frac{kQ}{r_2}\right)$$

4. Potential Difference Between Two Points

The potential difference between two points at distances r_1 and r_2 from a charge Q is:

$$\Delta V = V_1 - V_2 = \frac{kQ}{r_1} - \frac{kQ}{r_2}$$

A 2×10^{-9} C charge is moved from a distance of **0.5 m** to **1.5 m** from a charge of 5×10^{-6} C. Calculate the work done in moving the charge.

(Hint: Use $W = q\Delta V$, where $\Delta V = V_1 - V_2$)

Two charges, $Q_1 = 3 \times 10^{-6}$ C and $Q_2 = -2 \times 10^{-6}$ C, are placed **0.4 m** apart. Find the electric potential at the midpoint between them.

Four charges of $+5 \times 10^{-6}$ C are placed at the four corners of a square of side **0.4 m**. Find the **electric field at the center** of the square.

Coulomb's Law

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{r^2}$$

or using **Coulomb's constant** k:

$$\boxed{F = k\frac{Q_1 Q_2}{r^2}}$$

where:

- F = Electrostatic force (in Newtons, **N**)
- Q_1, Q_2 = Magnitudes of the two charges (in Coulombs, **C**)
- r = Distance between the charges (in meters, **m**)
- ε_0 = Permittivity of free space ($8.85 \times 10^{-12}\, C^2/N \cdot m^2$)
- k = Coulomb's constant ($9.0 \times 10^9\, N \cdot m^2/C^2$)

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Concept Learning Questions

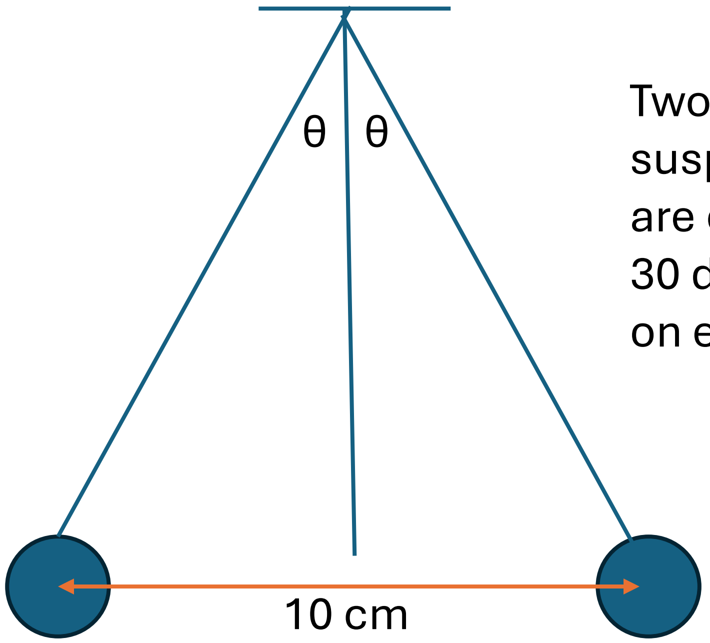
Two charges, $Q_1 = 5\,\mu C$ and $Q_2 = -3\,\mu C$, are placed $0.2\,m$ apart in vacuum. Calculate the electrostatic force between them.
(Take $k = 9.0 \times 10^9\,N \cdot m^2/C^2$)

If the distance between two charges is reduced to half its original value, how does the electrostatic force between them change? Consider the charges to be $Q_1 = 4\,\mu C$ and $Q_2 = -2\,\mu C$, and the initial distance $r = 0.3\,m$.

Three point charges are placed as follows:

- $Q_1 = 5\,\mu C$ at the origin,
- $Q_2 = -3\,\mu C$ at $(0, 0.4)\,m$,
- $Q_3 = 4\,\mu C$ at $(0, -0.3)\,m$.

  Calculate the net electrostatic force acting on Q_1 due to Q_2 and Q_3.



Two ideal spheres, each with a mass of 3 grams, are suspended from vertical strings of equal length. The spheres are equally charged and repel each other, making an angle of 30 degrees to the vertical. Find the magnitude of the charge on each sphere.

More questions.

01) Three charges are placed in **free space** at the following positions:

- $Q_1 = 2 \times 10^{-6}$ C at $(0, 0)$
- $Q_2 = -2 \times 10^{-6}$ C at $(0.3, 0)$
- $Q_3 = 5 \times 10^{-6}$ C at $(0.15, 0.15)$

Find the **electric potential at the point** $(0.2, 0.2)$.

Answer: $F_{\text{net}} = 1.8$ N at 74° above the x-axis

02)

A charge of $+6 \times 10^{-6}$ **C** is fixed at the origin. Another charge of $+2 \times 10^{-6}$ **C** is placed at $(0.2, 0)$ **m**.

1. Find the **electric field at (0.1, 0)**.
2. Where along the **x-axis** can a **third charge** be placed so that the net force on it is zero?

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Answer: $r = 0.12$ m for equilibrium

03) A **uniform electric field** of strength 5×10^4 **N/C** is directed along the **positive x-axis**.

1. A charge $+3 \times 10^{-6}$ **C** is moved from $(0, 0)$ **to** $(0.4, 0.3)$ **m**. Find the **work done** in moving the charge.
2. If the charge started from rest, find its final **kinetic energy**.

Answer for Work Done: 0.06 J

04) Two parallel plates are separated by **2 mm** and have a potential difference of **6000 V**. A **dielectric slab** with a relative permittivity $\varepsilon_r = 3.5$ is inserted, filling the gap completely.

1. Calculate the **electric field strength** between the plates before and after inserting the dielectric.
2. Determine the **capacitance** per unit area of the plates after inserting the dielectric.

Hint: Use

$$E = \frac{V}{d}, \quad C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

05) Two **conducting spheres** of radii **0.2 m** and **0.5 m** are charged with $Q_1 = 3 \times 10^{-6}$ **C** and $Q_2 = 6 \times 10^{-6}$ **C** respectively. The spheres are brought into **contact** and then separated.

1. Find the **final charge on each sphere**.
2. Calculate the **electric potential** on each sphere after separation.
3. Determine the **energy lost** during the process.

Answers:

Charge on Sphere 1: $Q_1 = 4 \times 10^{-6}$ C

Charge on Sphere 2: $Q_2 = 2.6 \times 10^{-6}$ C

When the spheres **are brought into contact**, charge redistributes until both have the **same potential**:

$$V_1 = V_2$$

Since the potential on a conducting sphere is given by:

$$V = \frac{Q}{R}$$

we set:

$$\frac{Q_{\text{final},1}}{R_1} = \frac{Q_{\text{final},2}}{R_2}$$

The total charge before contact is:

$$Q_{\text{total}} = Q_1 + Q_2 = (3 + 6) \times 10^{-6} = 9 \times 10^{-6}\ \text{C}$$

Charge distributes in proportion to their radii:

$$Q_{\text{final},1} = Q_{\text{total}} \times \frac{R_1}{R_1 + R_2}$$

$$Q_{\text{final},1} = (9 \times 10^{-6}) \times \frac{0.2}{0.2 + 0.5} = (9 \times 10^{-6}) \times \frac{0.2}{0.7}$$

$$Q_{\text{final},1} = 2.57 \times 10^{-6}\ \text{C}$$

Similarly, for Sphere 2:

$$Q_{\text{final},2} = Q_{\text{total}} \times \frac{R_2}{R_1 + R_2}$$

$$Q_{\text{final},2} = (9 \times 10^{-6}) \times \frac{0.5}{0.7}$$

$$Q_{\text{final},2} = 6.43 \times 10^{-6}\ \text{C}$$