IAL-A2 Physics

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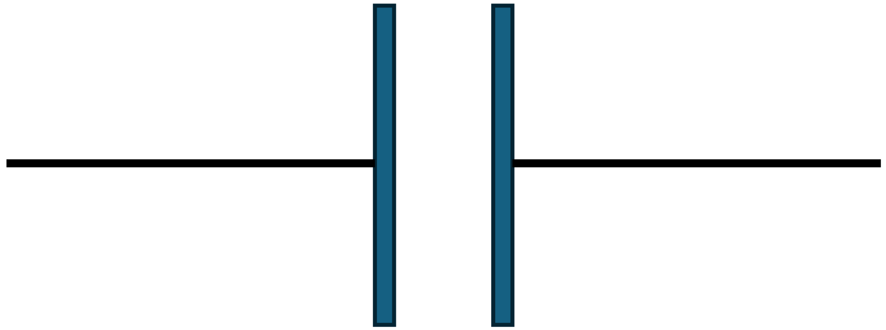
Capacitors

Capacitor

A **capacitor** is an electrical component that stores energy in the form of an **electric field** between two conductive plates separated by an **insulating material (dielectric)**.

$$C = \frac{Q}{V}$$

$$Q = CV$$

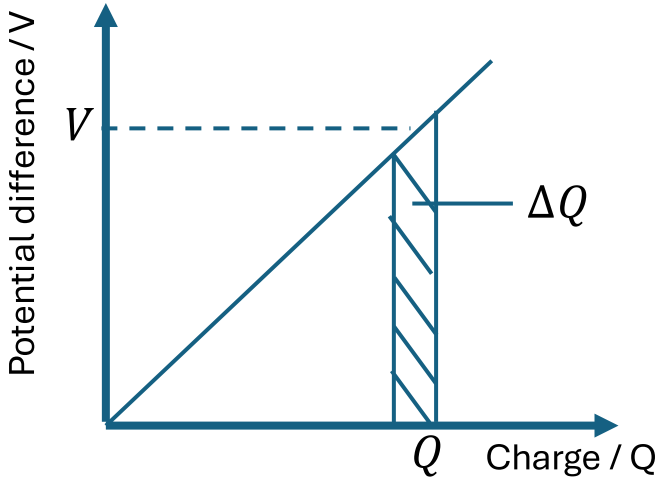


Circuit symbol

The ability of a capacitor to store charge is called capacitance (C), measured in farads (F).

Energy stored on a charged capacitor

$$E = \frac{1}{2}QV$$



$$V = \frac{Q}{C}$$

$$E = \frac{1}{2}Q(\frac{Q}{C})$$

$$Q = CV$$

$$E = \frac{1}{2}(CV)V$$

$$E = \frac{1}{2}CV^2$$

$$E = \frac{1}{2}(\frac{Q^2}{C})$$

Concept Learning Questions.

1) A capacitor with a capacitance of 10 μF is connected to a 12 V battery. Calculate the charge stored on the capacitor.

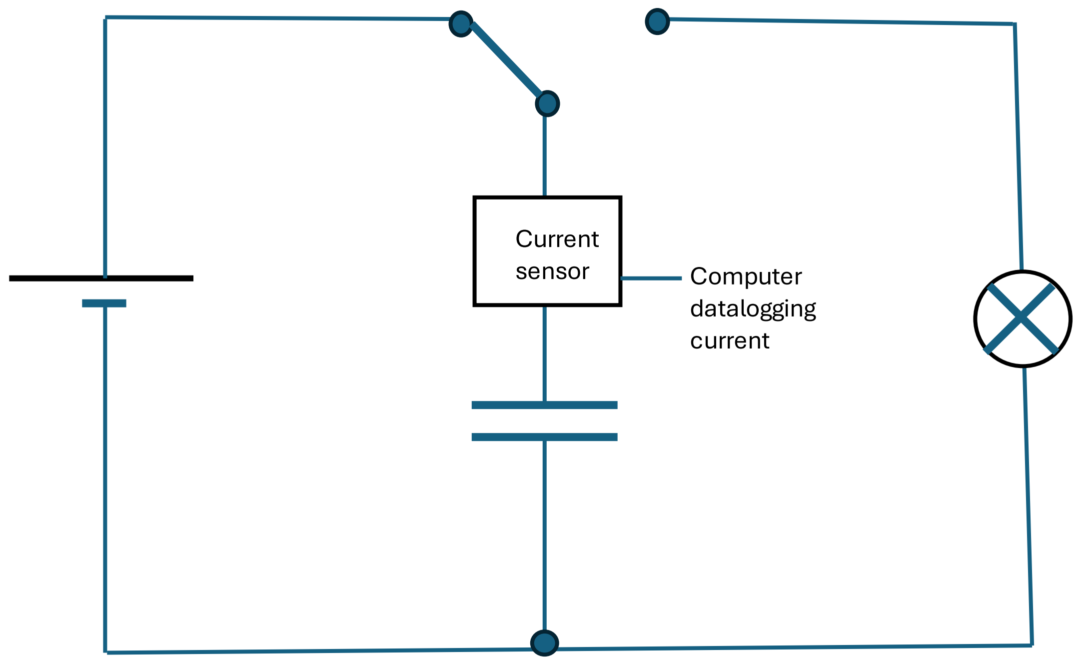
2) A capacitor stores a charge of 5 mC when connected to a 15 V potential difference. What is the capacitance of the capacitor?

3) A 2 μF capacitor is charged to a potential difference of 20 V. Calculate the energy stored in the capacitor.

4) A capacitor has a charge of 0.02 C and is connected to a potential difference of 10 V. Calculate the energy stored in the capacitor.

5) A capacitor with a capacitance of 3 μF stores a charge of 6 mC. Calculate the energy stored in the capacitor

Charging and discharging capacitors.



Advantages of using data logger with a current sensor.

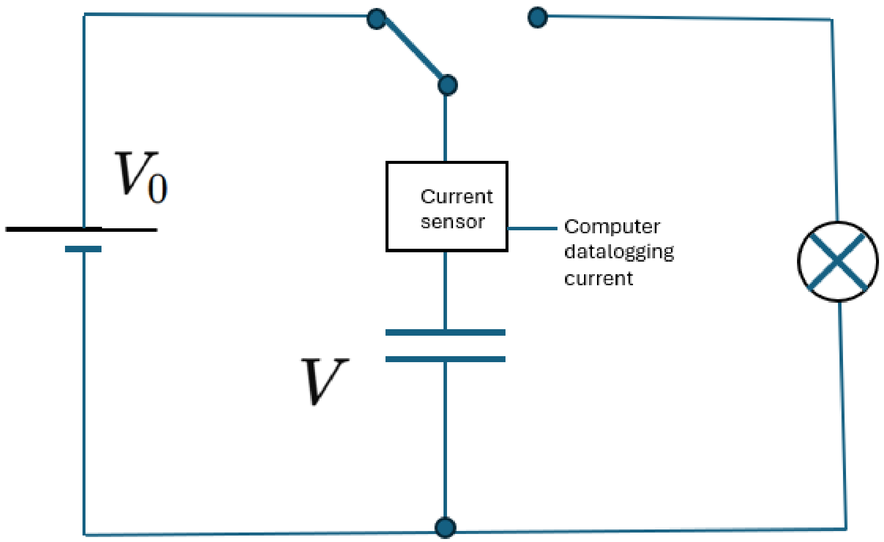
Greater accuracy and consistency

-Human error(such as misreading scales or parallax error) is eliminated.
-Data loggers use precise measurements and maintain consistent sampling intervals.

Continuous and automated recording.

-A data logger can collect current data around the clock, without needing a person to be present.
-Automatically saved in digital form and stored data can be easily transferred for analysis and graphing.

Charging the capacitor

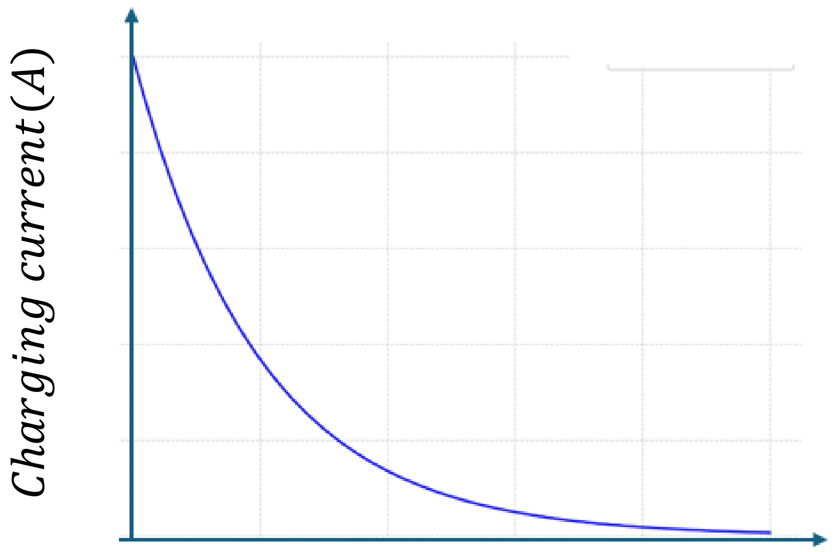


When a battery or voltage source is connected to the capacitor, it starts to push electrons onto one plate of the capacitor, creating a negative charge. The other plate becomes positively charged due to the loss of electrons.

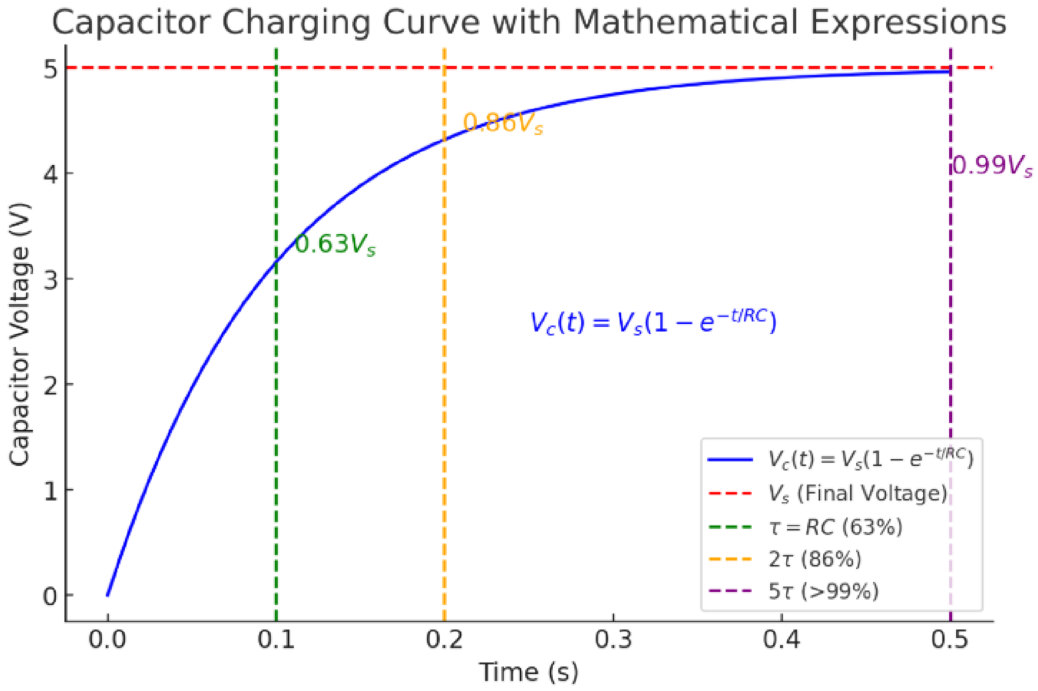
The voltage difference between the source and the capacitor causes current to flow through the circuit. This current charges the capacitor, and the charge on the capacitor increases over time.

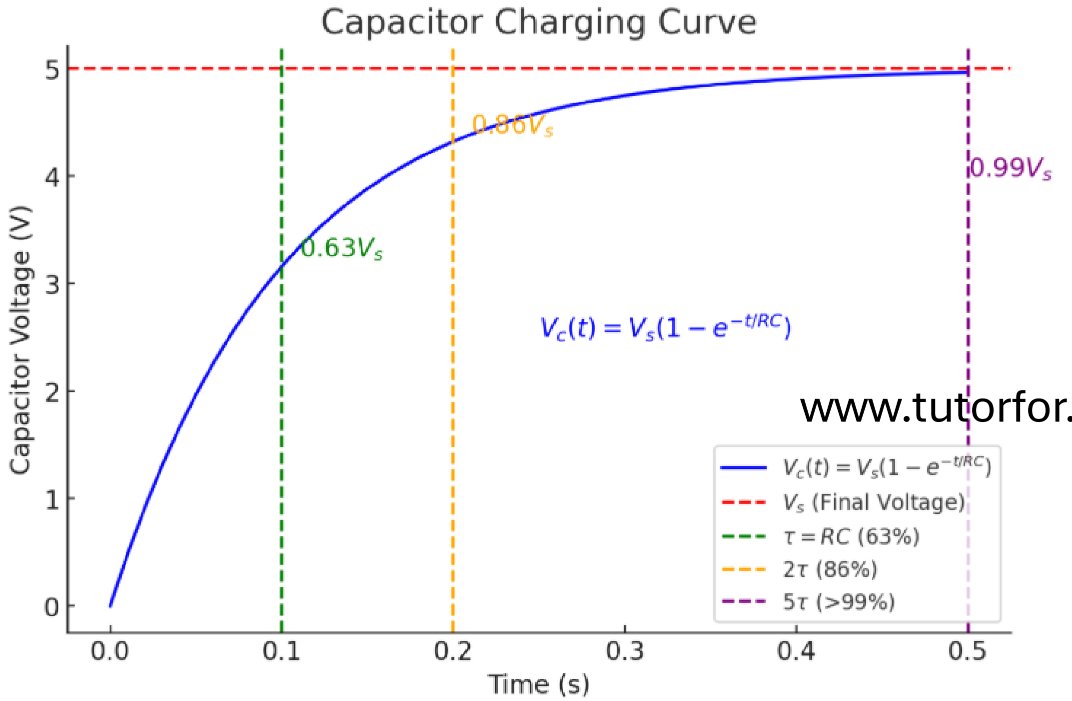
$$V = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

Charging graphs.



Once the capacitor is fully charged, the voltage across its plates equals the voltage of the battery, and the current stops flowing. At this point, the capacitor holds the maximum charge it can store.





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Time constant

The **time constant** (τ) is a key parameter in an RC circuit. It determines how fast the capacitor charges or discharges.

The time constant is given by:

$$\tau = R \cdot C$$

where:

- R = Resistance in ohms (Ω)
- C = Capacitance in farads (F)
- τ (tau) is measured in seconds (s)

The capacitor charging equation is:

$$V_c(t) = V_s \left(1 - e^{-t/\tau}\right)$$

At $t = \tau$, we substitute into the equation:

$$V_c(\tau) = V_s \left(1 - e^{-1}\right)$$

Since $e^{-1} \approx 0.367$, we get:

$$V_c(\tau) \approx 0.63 V_s$$

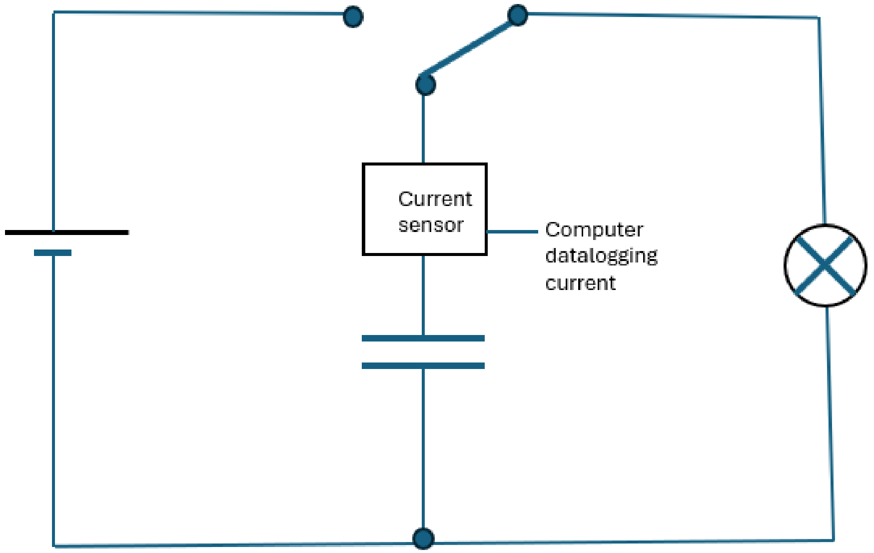
This means after **one time constant** (τ), the capacitor is **63% charged**.

Each multiple of τ follows the exponential rule:

Time t	Voltage $V_c(t)$
0	$0V$ (0%)
τ	$0.63V_s$ (63%)
2τ	$0.86V_s$ (86%)
3τ	$0.95V_s$ (95%)
5τ	$0.99V_s$ (99%)

After **5τ**, the capacitor is practically **fully charged**.

Discharging Capacitor



When the capacitor is connected to the resistor (like a lamp), the stored charge begins to flow through the resistor. As the capacitor discharges, electrons move from the negative plate of the capacitor through the resistor to the positive plate.

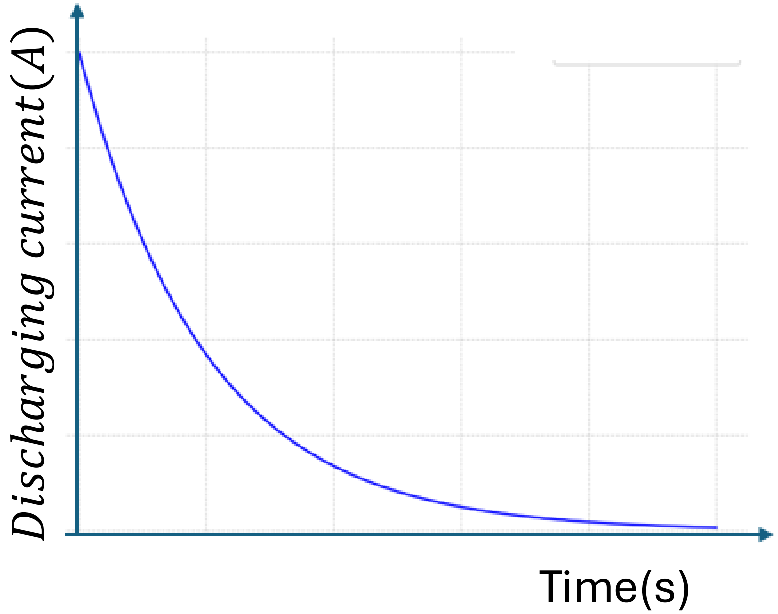
As the capacitor discharges, the charge on its plates decreases.

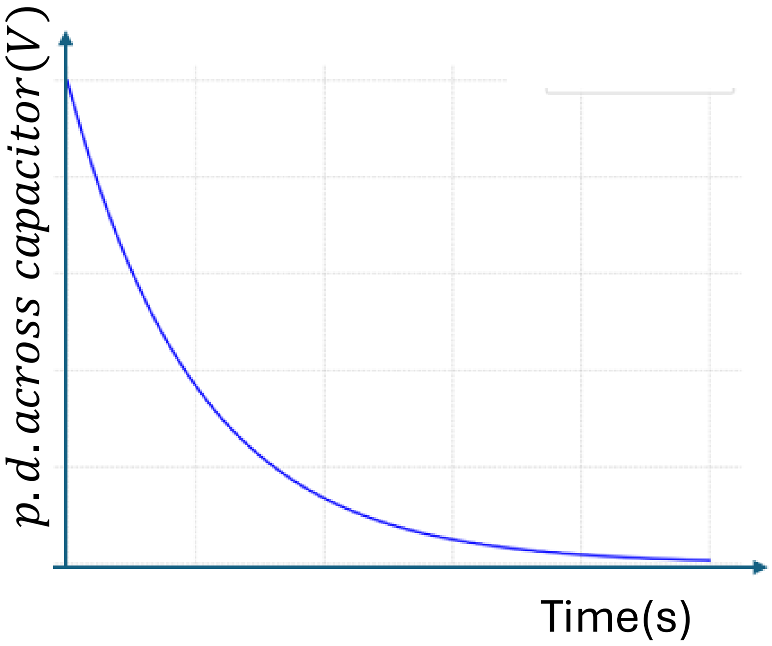
$$Q = Q_0 e^{-\frac{t}{RC}}$$

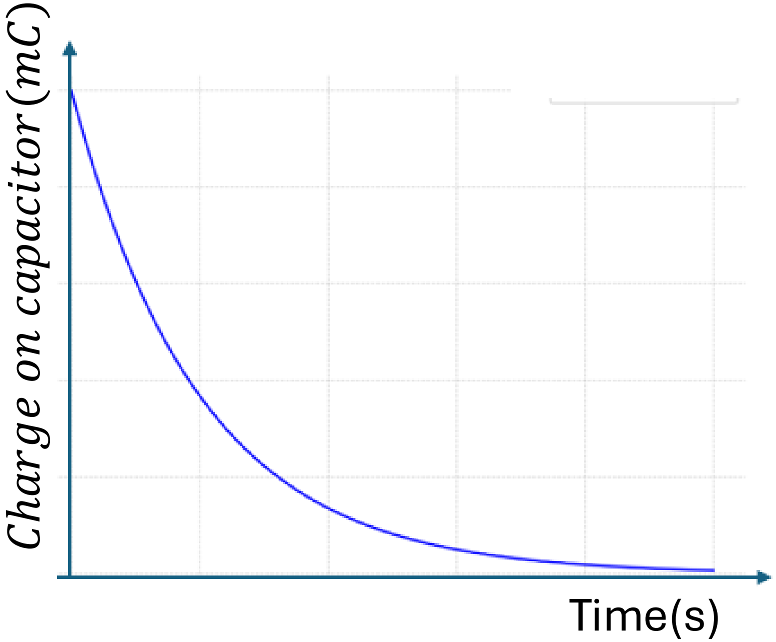
This means the voltage across the capacitor also decreases. The current decreases over time as the voltage across the capacitor decreases.

$$V = V_0 e^{-\frac{t}{RC}}$$

Discharging graphs







$$Q = Q_0 e^{-\frac{t}{RC}}$$

$$V = V_0 e^{-\frac{t}{RC}}$$

$$I = I_0 e^{-\frac{t}{RC}}$$

When the capacitor discharges, its voltage $V_c(t)$ follows:

$$V_c(t) = V_s e^{-t/\tau}$$

where:

- V_s is the initial voltage of the capacitor before discharge.
- $e^{-t/\tau}$ represents the exponential decay factor.

At $t = \tau$:

$$V_c(\tau) = V_s e^{-1} \approx 0.37 V_s$$

So, after **one time constant** (τ), the capacitor retains only **37%** of its initial voltage.

Each multiple of τ follows the exponential decay rule:

Time t	Voltage $V_c(t)$
0	V_s (100%)
τ	$0.37V_s$ (37%)
2τ	$0.135V_s$ (13.5%)
3τ	$0.05V_s$ (5%)
5τ	$\approx 0V$ (<1%)

After **5τ**, the capacitor is practically **fully discharged**.

Mathematics knowledge

$$\ln(x) = y \quad \text{if and only if} \quad e^y = x$$

Where:

- $\ln(x)$ is the natural logarithm of x,
- e is Euler's number (approximately 2.71828),
- $x > 0$ (the argument of the natural logarithm must be positive).

$$\ln(1) = 0 \quad \text{because} \quad e^0 = 1$$

$$\ln(e) = 1 \quad \text{because} \quad e^1 = e$$

$$\ln(xy) = \ln(x) + \ln(y)$$

$$\ln\left(\frac{x}{y}\right) = \ln(x) - \ln(y)$$

$$\ln(x^n) = n\ln(x)$$

Capacitor calculus

In a simple RC circuit, the voltage across the resistor V_R and the voltage across the capacitor V_C must sum to zero:

$$V_R + V_C = 0$$

Since the resistor and the capacitor are connected in series, the voltage across the resistor V_R is given by Ohm's law:

$$V_R = I \cdot R$$

And the voltage across the capacitor V_C is related to the charge Q on the capacitor:

$$V_C = \frac{Q}{C}$$

$$I \cdot R + \frac{Q}{C} = 0$$

The current I in a circuit is the rate of change of charge with respect to time:

$$I = -\frac{dQ}{dt}$$

(The negative sign appears because the charge is decreasing as the capacitor discharges.)

Substituting this into the KVL equation:

$$-\frac{dQ}{dt} \cdot R + \frac{Q}{C} = 0$$

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

$$\frac{dQ}{Q} = -\frac{1}{RC}dt$$

$$\int_{Q_0}^{Q} \frac{1}{Q}dQ = -\int_0^t \frac{1}{RC}dt$$

$$\int_0^t \frac{1}{RC}dt = \frac{t}{RC}$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

$$Q = Q_0 e^{-\frac{t}{RC}}$$

Concept Learning Questions.

A **100 μF** capacitor is connected to a **1 kΩ** resistor. Calculate the time constant (τ) of the circuit.

Answer: 0.1 s

A capacitor is connected to a **12V** supply through a **2kΩ** resistor. If the time constant of the circuit is **0.5 seconds**, find the capacitor voltage after **1 second**.

Answer: 10.38 V

A capacitor charged to **10V** is allowed to discharge through a **500Ω** resistor. If the time constant is **0.2 seconds**, what will be the voltage after **0.4 seconds**?

Answer: 1.35 V

How long does it take for a capacitor to charge up to **90%** of the supply voltage if the time constant is **0.3 seconds**?

Answer: 0.69 s

A capacitor is initially charged to **15V** and discharges through a **3kΩ** resistor. If the capacitance is **200 μF**, how long will it take for the voltage to drop below **2V**?

Answer: 1.21 s